DATA MINING 2
Exercises – KNN, Naïve Bayes

Riccardo Guidotti, Salvatore Citraro

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UNIVERSITÀ DI PISA

K-NN

k-Nearest Neighbor Classifier

A medical expert is going to build up a case-based reasoning system for diagnosis tasks. Cases correspond to individual persons where the case problem parts are made up of a number of features describing possible symptoms and the solution parts represent the diagnosis (classification of disease). The case base contains the seven cases provided in the table below.

Training	Fever	Vomiting	Diarrhea	Shivering	Classification
c_1	no	no	no	no	healty (H)
c_2	average	no	no	no	influenza (I)
c_3	high	no	no	yes	influenza (I)
c_4	high	yes	yes	no	salmonella poisoning (S)
c_5	average	no	yes	no	salmonella poisoning (S)
c_6	no	yes	yes	no	bowel inflammation (B)
c_7	average	yes	yes	no	bowel inflammation (B)

Similarity provided by an expert

sim_F

q \ c	no	avg	high
no	1.0	0.7	0.2
avg	0.5	1.0	0.8
high	0.0	0.3	1.0

$sim_V = sim_D = sim_{Sh}$

q \	yes	no
yes	1.0	0.0
no	0.2	1.0

Weights

$w_F = 0.3$

$w_V = 0.2$

$W_D = 0.2$

$w_{Sh} = 0.3$

Classify the new instance q = (high; no; no; no) by applying the KNN algorithm with K=1,2,3

Calculate the similarity between all cases from the case base and the new instance q = (high; no; no; no)

c1 = (no; no; no; no):

Sim(q; c1) = 0.3*0.0 + 0.2 *1.0 + 0.2*1.0 + 0.3* 1.0 = 0.70

c2 = (average; no; no; no):

Sim(q; c2) = 0.3* 0.3 + 0.2 *1.0 + 0.2*1.0 + 0.3*1.0 = 0.79

c3 = (high; no; no; yes)

Sim(q; c3) = 0.3*1.0 + 0.2*1.0 + 0.2*1.0 + 0.3*0.2 = 0.76

c4 = (high; yes; yes; no):

Sim(q; c4) = 0.3*1.0 + 0.2*0.2 + 0.2*0.2 + 0.3*1.0 = 0.68

c5 = (average; no; yes; no):

Sim(q; c5) = 0.3*0.3 + 0.2*1.0 + 0.2*0.2 + 0.3*1.0 = 0.63

c6 = (no; yes; yes; no):

Sim(q; c6) = 0.3*0.0 + 0.2*0.2 + 0.2*0.2 + 0.3*1.0 = 0.28

c7 = (average; yes; yes; no):

Sim(q; c7) = 0.3*0.3 + 0.2*0.2 + 0.2*0.2 + 0.3*1.0 = 0.47

sim_F

q \ c	no	avg	high
no	1.0	0.7	0.2
avg	0.5	1.0	0.8
high	0.0	0.3	1.0

$sim_V = sim_D = sim_{Sh}$

q \	yes	no
yes	1.0	0.0
no	0.2	1.0

Weights

$w_F = 0.3$

$w_V = 0.2$

$W_D = 0.2$

$w_{Sh} = 0.3$

KNN Classification for K=1

c1 = (no; no; no; no):

Sim(q; c1) = 0.3*0.0 + 0.2 *1.0 + 0.2*1.0 + 0.3* 1.0 = 0.70

c2 = (average; no; no; no):

Sim(q; c2) = 0.3* 0.3 + 0.2 *1.0 + 0.2*1.0 + 0.3*1.0 = 0.79

c3 = (high; no; no; yes)

Sim(q; c3) = 0.3*1.0 + 0.2*1.0 + 0.2*1.0 + 0.3*0.2 = 0.76

c4 = (high; yes; yes; no):

Sim(q; c4) = 0.3*1.0 + 0.2*0.2 + 0.2*0.2 + 0.3*1.0 = 0.68

c5 = (average; no; yes; no):

Sim(q; c5) = 0.3*0.3 + 0.2*1.0 + 0.2*0.2 + 0.3*1.0 = 0.63

c6 = (no; yes; yes; no):

Sim(q; c6) = 0.3*0.0 + 0.2*0.2 + 0.2*0.2 + 0.3*1.0 = 0.28

c7 = (average; yes; yes; no):

Sim(q; c7) = 0.3*0.3 + 0.2*0.2 + 0.2*0.2 + 0.3*1.0 = 0.47

sim_F

q \ c	no	avg	high
no	1.0	0.7	0.2
avg	0.5	1.0	0.8
high	0.0	0.3	1.0

Weights

w_F=0.3

w_V=0.2

W_D=0.2

w_{Sh}=0.3

Class: Influenza

KNN Classification for K=2

c1 = (no; no; no; no):

Sim(q; c1) = 0.3*0.0 + 0.2 *1.0 + 0.2*1.0 + 0.3* 1.0 = 0.70

c2 = (average; no; no; no):

Sim(q; c2) = 0.3* 0.3 + 0.2 *1.0 + 0.2*1.0 + 0.3*1.0 = 0.79

c3 = (high; no; no; yes):

Sim(q; c3) = 0.3*1.0 + 0.2*1.0 + 0.2*1.0 + 0.3*0.2 = 0.76

c4 = (high; yes; yes; no):

Sim(q; c4) = 0.3*1.0 + 0.2*0.2 + 0.2*0.2 + 0.3*1.0 = 0.68

c5 = (average; no; yes; no):

Sim(q; c5) = 0.3*0.3 + 0.2*1.0 + 0.2*0.2 + 0.3*1.0 = 0.63

c6 = (no; yes; yes; no):

Sim(q; c6) = 0.3*0.0 + 0.2*0.2 + 0.2*0.2 + 0.3*1.0 = 0.28

c7 = (average; yes; yes; no):

Sim(q; c7) = 0.3*0.3 + 0.2*0.2 + 0.2*0.2 + 0.3*1.0 = 0.47

sim_F

q \ c	no	avg	high
no	1.0	0.7	0.2
avg	0.5	1.0	0.8
high	0.0	0.3	1.0

Weights

w_F=0.3

w_V=0.2

W_D=0.2

w_{Sh}=0.3

C2: Influenza

C3: Influenza

Class: Influenza

KNN Classification for K=3

c1 = (no; no; no; no):

Sim(q; c1) = 0.3*0.0 + 0.2 *1.0 + 0.2*1.0 + 0.3* 1.0 = 0.70

c2 = (average; no; no; no):

Sim(q; c2) = 0.3* 0.3 + 0.2 *1.0 + 0.2*1.0 + 0.3*1.0 = 0.79

c3 = (high; no; no; yes):

Sim(q; c3) = 0.3*1.0 + 0.2*1.0 + 0.2*1.0 + 0.3*0.2 = 0.76

c4 = (high; yes; yes; no):

Sim(q; c4) = 0.3*1.0 + 0.2*0.2 + 0.2*0.2 + 0.3*1.0 = 0.68

c5 = (average; no; yes; no):

Sim(q; c5) = 0.3*0.3 + 0.2*1.0 + 0.2*0.2 + 0.3*1.0 = 0.63

c6 = (no; yes; yes; no):

Sim(q; c6) = 0.3*0.0 + 0.2*0.2 + 0.2*0.2 + 0.3*1.0 = 0.28

c7 = (average; yes; yes; no):

Sim(q; c7) = 0.3*0.3 + 0.2*0.2 + 0.2*0.2 + 0.3*1.0 = 0.47

sim_F

q \ c	no	avg	high
no	1.0	0.7	0.2
avg	0.5	1.0	0.8
high	0.0	0.3	1.0

Weights

w_F=0.3

w_V=0.2

W_D=0.2

w_{Sh}=0.3

C1: healty

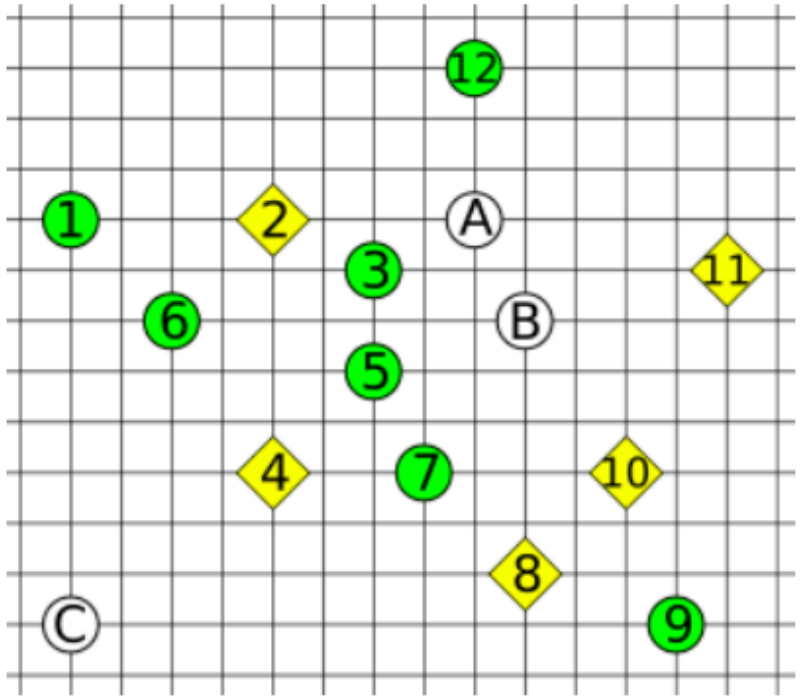
C2: Influenza

C3: Influenza

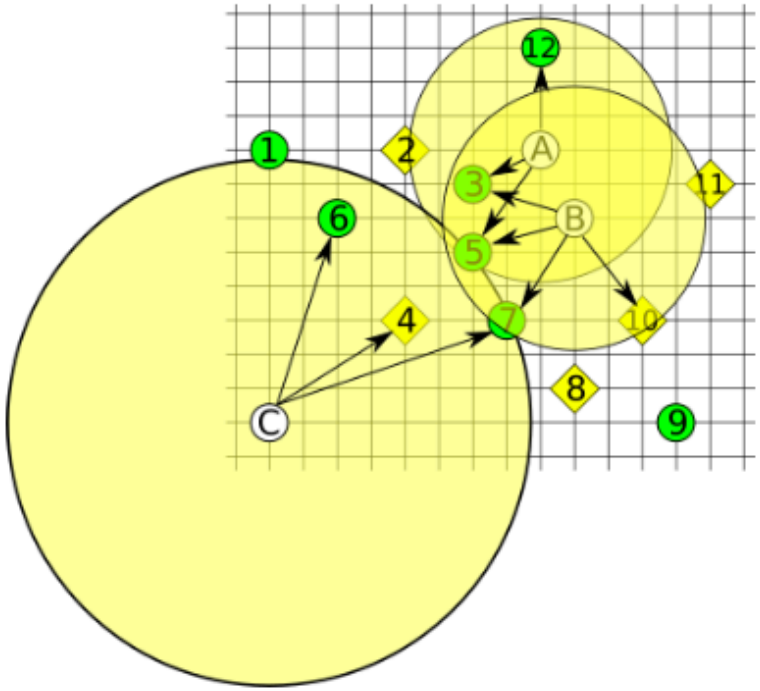
Class: Influenza

b) k-NN (**3 points**)

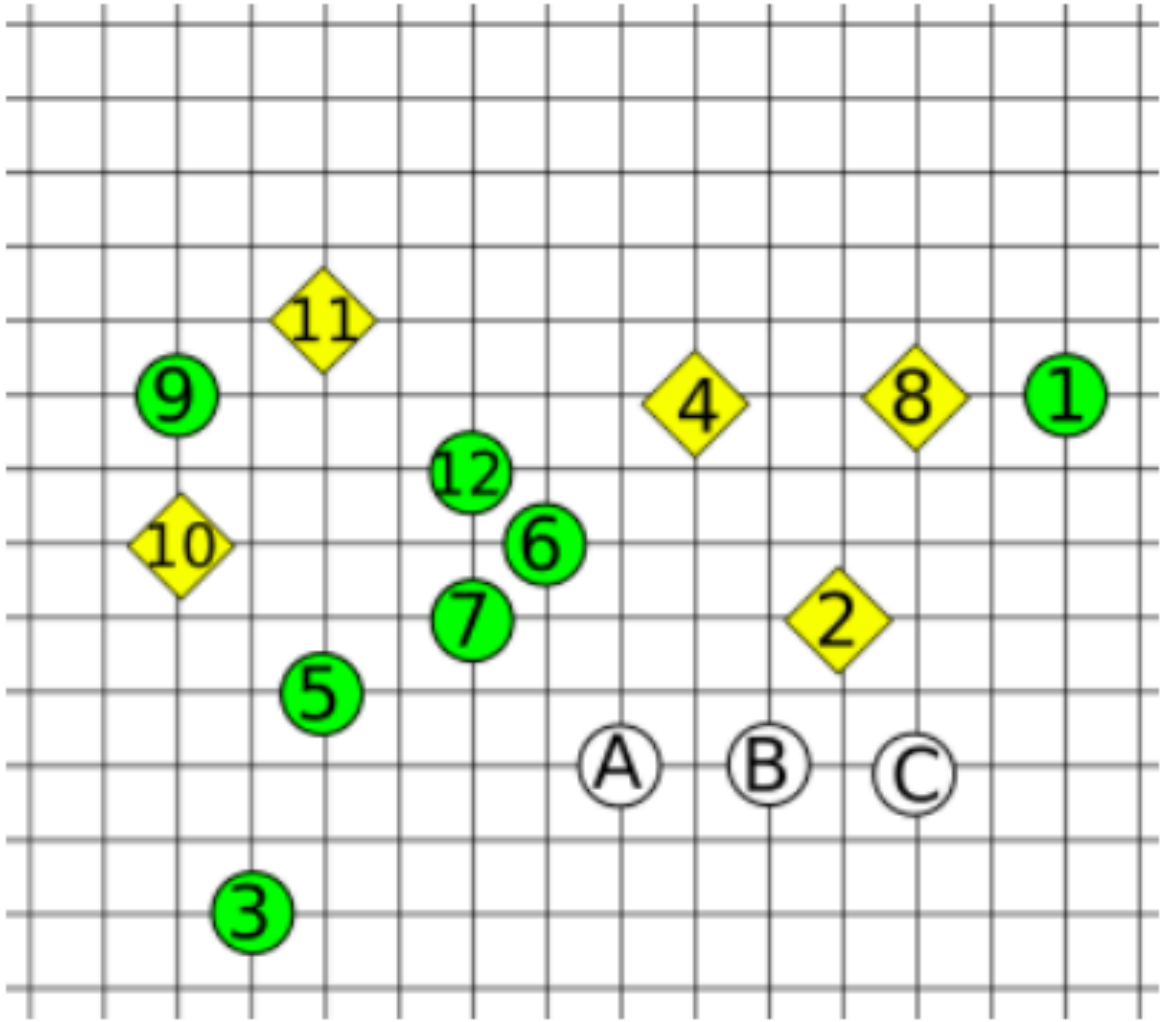
Given the training set on the right, composed of elements numbered from 1 to 12, and labelled as circles and diamonds, use it to classify the remaining 3 elements (letters A, B and C) using a k-NN classifier with k=3. For each point to classify, list the points of the dataset that belong to its k-NN set.
Notice: A, B and C belong to the test set, not to the training set. Also, the Euclidean distance should be used.



Answer:
kNN(A) = {3, 5, 12} → CIRCLE
kNN(B) = { 3, 5, 7, 10 } → CIRCLE
kNN(C) = { 4, 6, 7 } → CIRCLE



Given the training set on the right, composed of elements numbered from 1 to 12, and labelled as circles and diamonds, use it to classify the remaining 3 elements (letters A, B and C) using a k-NN classifier with k=3.
For each point to classify, list the points of the dataset that belong to its k-NN set.
Notice: A, B and C belong to the test set, not to the training set. Also, the Euclidean distance should be used.



Naïve Bayes

Play-tennis example. estimating $P(x_i|C)$

Outlook	Temperature	Humidity	Windy	Class
sunny	hot	high	false	N
sunny	hot	high	true	N
overcast	hot	high	false	P
rain	mild	high	false	P
rain	cool	normal	false	P
rain	cool	normal	true	N
overcast	cool	normal	true	P
sunny	mild	high	false	N
sunny	cool	normal	false	P
rain	mild	normal	false	P
sunny	mild	normal	true	P
overcast	mild	high	true	P
overcast	hot	normal	false	P
rain	mild	high	true	N

P(p) = 9/14
P(n) = 5/14

outlook	
P(sunny|p) =	**P(sunny|n) =**
P(overcast|p) =	**P(overcast|n) =**
P(rain|p) =	**P(rain|n) =**
temperature	
P(hot|p) =	**P(hot|n) =**
P(mild|p) =	**P(mild|n) =**
P(cool|p) =	**P(cool|n) =**
humidity	
P(high|p) =	**P(high|n) =**
P(normal|p) =	**P(normal|n) =**
windy	
P(true|p) =	**P(true|n) =**
P(false|p) =	**P(false|n) =**

Play-tennis example. estimating $P(x_i|C)$

Outlook	Temperature	Humidity	Windy	Class
sunny	hot	high	false	N
sunny	hot	high	true	N
overcast	hot	high	false	P
rain	mild	high	false	P
rain	cool	normal	false	P
rain	cool	normal	true	N
overcast	cool	normal	true	P
sunny	mild	high	false	N
sunny	cool	normal	false	P
rain	mild	normal	false	P
sunny	mild	normal	true	P
overcast	mild	high	true	P
overcast	hot	normal	false	P
rain	mild	high	true	N

P(p) = 9/14
P(n) = 5/14

outlook	
P(sunny|p) = 2/9	**P(sunny|n) = 3/5**
P(overcast|p) = 4/9	**P(overcast|n) = 0**
P(rain|p) = 3/9	**P(rain|n) = 2/5**
temperature	
P(hot|p) = 2/9	**P(hot|n) = 2/5**
P(mild|p) = 4/9	**P(mild|n) = 2/5**
P(cool|p) = 3/9	**P(cool|n) = 1/5**
humidity	
P(high|p) = 3/9	**P(high|n) = 4/5**
P(normal|p) = 6/9	**P(normal|n) = 1/5**
windy	
P(true|p) = 3/9	**P(true|n) = 3/5**
P(false|p) = 6/9	**P(false|n) = 2/5**

Play-tennis example. estimating $P(x_i|C)$

P(p) = 9/14
P(n) = 5/14

Outlook	Temeprature	Humidity	Windy	Class
rain	hot	high	false	?

outlook	
P(sunny|p) = 2/9	P(sunny|n) = 3/5
P(overcast|p) = 4/9	P(overcast|n) = 0
P(rain|p) = 3/9	P(rain|n) = 2/5
temperature	
P(hot|p) = 2/9	P(hot|n) = 2/5
P(mild|p) = 4/9	P(mild|n) = 2/5
P(cool|p) = 3/9	P(cool|n) = 1/5
humidity	
P(high|p) = 3/9	P(high|n) = 4/5
P(normal|p) = 6/9	P(normal|n) = 1/5
windy	
P(true|p) = 3/9	P(true|n) = 3/5
P(false|p) = 6/9	P(false|n) = 2/5

$P(X|p)\cdot P(p) =$

$P(X|n)\cdot P(n) =$

Play-tennis example. estimating $P(x_i|C)$

P(p) = 9/14
P(n) = 5/14

Outlook	Temeprature	Humidity	Windy	Class
rain	hot	high	false	N

outlook	
P(sunny|p) = 2/9	**P(sunny|n) = 3/5**
P(overcast|p) = 4/9	**P(overcast|n) = 0**
P(rain|p) = 3/9	**P(rain|n) = 2/5**
temperature	
P(hot|p) = 2/9	**P(hot|n) = 2/5**
P(mild|p) = 4/9	**P(mild|n) = 2/5**
P(cool|p) = 3/9	**P(cool|n) = 1/5**
humidity	
P(high|p) = 3/9	**P(high|n) = 4/5**
P(normal|p) = 6/9	**P(normal|n) = 1/5**
windy	
P(true|p) = 3/9	**P(true|n) = 3/5**
P(false|p) = 6/9	**P(false|n) = 2/5**

P(X|p)·P(p) = P(rain|p)·P(hot|p)·P(high|p)·P(false|p)·P(p) = 3/9 · 2/9 · 3/9 · 6/9 · 9/14 = 0.010582

P(X|n)·P(n) = P(rain|n)·P(hot|n)·P(high|n)·P(false|n)·P(n) = 2/5 · 2/5 · 4/5 · 2/5 · 5/14 = 0.018286

Example of Naïve Bayes Classifier

Name	Give Birth	Can Fly	Live in Water	Have Legs	Class
human	yes	no	no	yes	mammals
python	no	no	no	no	non-mammals
salmon	no	no	yes	no	non-mammals
whale	yes	no	yes	no	mammals
frog	no	no	sometimes	yes	non-mammals
komodo	no	no	no	yes	non-mammals
bat	yes	yes	no	yes	mammals
pigeon	no	yes	no	yes	non-mammals
cat	yes	no	no	yes	mammals
leopard shark	yes	no	yes	no	non-mammals
turtle	no	no	sometimes	yes	non-mammals
penguin	no	no	sometimes	yes	non-mammals
porcupine	yes	no	no	yes	mammals
eel	no	no	yes	no	non-mammals
salamander	no	no	sometimes	yes	non-mammals
gila monster	no	no	no	yes	non-mammals
platypus	no	no	no	yes	mammals
owl	no	yes	no	yes	non-mammals
dolphin	yes	no	yes	no	mammals
eagle	no	yes	no	yes	non-mammals

A: attributes

M: mammals

N: non-mammals

$$P(A \mid M) = \frac{6}{7} \times \frac{6}{7} \times \frac{2}{7} \times \frac{2}{7} = 0.06$$

$$P(A \mid N) = \frac{1}{13} \times \frac{10}{13} \times \frac{3}{13} \times \frac{4}{13} = 0.0042$$

$$P(A \mid M)P(M) = 0.06 \times \frac{7}{20} = 0.021$$

$$P(A \mid N)P(N) = 0.004 \times \frac{13}{20} = 0.0027$$

Give Birth	Can Fly	Live in Water	Have Legs	Class
yes	no	yes	no	?

P(A|M)P(M) > P(A|N)P(N)

=> Mammals

a) Naive Bayes (**3 points**)

Given the training set below, build a Naive Bayes classification model (i.e. the corresponding table of probabilities) using (i) the normal formula and (ii) using Laplace formula. What are the main effects of Laplace on the models?

A	B	class
no	green	N
no	red	Y
yes	green	N
no	red	N
no	red	Y
no	green	Y
yes	green	N

Answer:

Normal

	Y	N
	3	4
	A | Y	A | N
yes	0	2
no	3	2
	B | Y	B | N
green	1	3
red	2	1

	Y	N
	0.43	0.57
	A | Y	A | N
yes	0.00	0.50
no	1.00	0.50
	B | Y	B | N
green	0.33	0.75
red	0.67	0.25

Laplace

	Y	N
	3	4
	A | Y	A | N
yes	0	2
no	3	2
	B | Y	B | N
green	1	3
red	2	1

	Y	N
	0.43	0.57
	A | Y	A | N
yes	0.20	0.50
no	0.80	0.50
	B | Y	B | N
green	0.40	0.67
red	0.60	0.33

a) Naive Bayes (**3 points**)

Given the training set on the left, build a Naive Bayes classification model and apply it to the test set on the right.

SCORE	FIRST-TRY	FACULTY	class
good	no	science	Y
medium	yes	science	N
bad	yes	science	N
bad	yes	humanities	Y
good	no	humanities	N
good	no	science	Y
medium	no	humanities	Y

SCORE	FIRST-TRY	FACULTY	class
bad	no	humanities	
good	yes	science	
medium	yes	humanities	